

**QUIZ #1 – Solutions**  
**Each problem is worth 5 points**

**15 points total**

1.

For the given function,

$$\begin{aligned} x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 4y &= x^2 \left[ \frac{2C_1}{x^2} \sin(2 \ln x) - \frac{4C_1}{x^2} \cos(2 \ln x) - \frac{2C_2}{x^2} \cos(2 \ln x) - \frac{4C_2}{x^2} \sin(2 \ln x) \right] \\ &\quad + x \left[ -\frac{2C_1}{x} \sin(2 \ln x) + \frac{2C_2}{x} \cos(2 \ln x) \right] + 4[C_1 \cos(2 \ln x) + C_2 \sin(2 \ln x) + 1/4] \\ &= 1. \end{aligned}$$

2.

Integration with respect to  $x$  gives a general solution  $y(x) = 2x^3 + x^2 + C$ .

3.

We separate this equation,  $\left(\frac{y-1}{y}\right) dy = \left(\frac{x+1}{x}\right) dx$ , (provided  $y \neq 0$ ). A one-parameter family of solutions defined implicitly by  $y - \ln|y| = x + \ln|x| + C$ . When we take exponentials,  $xy = De^{y-x}$ , where  $D = \pm e^{-C}$ . To satisfy  $y(1) = 2$ , we must have  $(1)(2) = De^{2-1}$ . Thus,  $D = 2/e$ , and  $xy = 2e^{y-x-1}$ .